Learning Shortest Paths in Word Graphs[*]

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Abstract

In this paper we briefly sketch our work on text summarisation using compression graphs. The task is described as follows: Given a set of related sentences describing the same event, we aim at generating a single sentence that is simply structured, easily understandable, and minimal in terms of the number of words/tokens. Traditionally, sentence compression deals with finding the shortest path in word graphs in an unsupervised setting. The major drawback of this approach is the use of manually crafted heuristics for edge weights. By contrast, we cast sentence compression as a structured prediction problem. Edges of the compression graph are represented by features drawn from adjacent nodes so that corresponding weights are learned by a generalised linear model. Decoding is performed in polynomial time by a generalised shortest path algorithm using loss augmented inference. We report on preliminary results on artificial and real world data.

1 Introduction

In this paper we study the intelligent summarisation of related sentences to quickly serve information needs of users. Given a collection of sentences dealing with the same real-world event, we aim at generating a single sentence that is (i) a summarisation of the input sentences, (ii) simply structured and easily understandable, and (iii) minimal in terms of the number of words/tokens. The input sentences are represented as a word graph [Filippova, 2010], where words are identified with nodes and directed edges connect adjacent words in at least one sentence, and the output summary is thus a path in the graph fulfilling conditions (i-iii). In this paper, we cast sentence compression as learning a mapping from word graphs to their shortest paths. Edges of the graphs are labeled with costs and the shortest path realises the lowest possible costs from a start to an end node.

Learning mappings between arbitrary structured and interdependent input and output spaces challenges the standard model of learning a mapping from independently drawn instances to a small set of labels. For capturing the involved dependencies it is helpful to represent inputs $\mathbf{x} \in \mathcal{X}$ and outputs $\mathbf{y} \in \mathcal{Y}$ in a joint feature representation. The standard approach to learn to predict structured outputs seeks the most likely output structure given an input. By contrast, we aim at finding the shortest and thus minimal path that leads from a start to an end node of the compression graph. Therefore the task is rephrased as finding a function $f : \mathcal{X} \times \mathcal{Y} \to \Re$ such that

$$\hat{\mathbf{y}} = \underset{\mathbf{y}}{\operatorname{argmin}} f(\mathbf{x}, \mathbf{y}) \qquad (1)$$

is the desired output for any input $\mathbf{x}$ [Tsochantaridis *et al.*, 2005; Taskar *et al.*, 2004]. The function f is a linear model in a joint space $\Phi(\mathbf{x}, \mathbf{y})$ of input and output variables and the computation of the argmin is performed by an appropriate decoding strategy such as a shortest-path algorithm.

The remainder is organised as follows. Section 2 introduces preliminaries. Our main contribution is presented in 3. We briefly discuss empirical results in Section 4 and Section 5 concludes.

2 Preliminaries

2.1 Related Work

Barzilay and Lee [Barzilay and Lee, 2003] study sentence compression using dependency trees. Aligned trees are represented by a lattice from which a compression sentence is extracted by an entropy-based criterion over all possible traversals of the lattice. Wan et al. at [Wan *et al.*, 2007] use a language model in combination with maximum spanning trees to rank candidate aggregations that satisfy grammatical constrains.

While the previous approaches to multi-sentence compression are based on syntactic parsing of the sentences, word graph approaches have been proposed, that do not make use of dependency trees or other linguistic concepts. Filippova [Filippova, 2010] casts the problem as finding the shortest path in directed word graphs, where each node is a unique word and directed edges represent the word ordering in the original sentences. The costs of these edges are given by a heuristic that is based on word frequencies. Recently, Boudin and Morin [Boudin and Morin, 2013] propose a re-ranking scheme to identify summarising sentences that contain many keyphrases. The underlying idea is that representative key phrases for a given topic give rise to more informative aggregations.

2.2 Word Graphs and Shortest Paths

Word graphs intend to build a non-redundant representation for possibly redundant sequences by merging identical observations. From a collection of related sentences we iteratively construct a word graph by adding sentences one-by-one as follows: We begin with an empty graph and add the first sentence, where every word in the sentence becomes a

[*]This paper is a short version of [Tzouridis and Brefeld, 2013].

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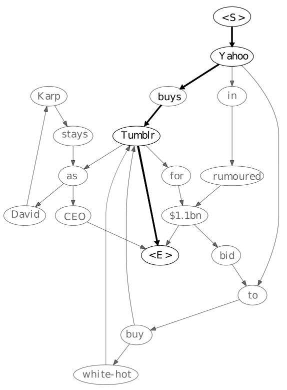


Figure 1: The word graph constructed from the sentences: "Yahoo in rumoured $1.1bn bid to buy white-hot Tumblr", "Yahoo buys Tumblr as David Karp stays as CEO", "Yahoo to buy Tumblr for $1.1bn". The corresponding shortest path is highlighted.

node and a directed edge connects nodes of adjacent words. Words from the next sentences are incorporated by creating a new node for the word or by mapping the word to the corresponding already existing node. A directed edge is inserted to connect the word to its predecessor. We continue until all sentences are incorporated.

Auxiliary words indicating the start (e.g, x_s) and the end (e.g., x_e) of the sentence are added to the sentences. The sketched procedure merges identical words but preserves the structure of the sentences along the contained paths and the original sentences can often be reconstructed from the compressed representation. Fig. 1 shows related sentences and the corresponding word graph.

The described construction gives us a directed weighted graph $\mathbf{x} = (N, E)$, where N is the set of nodes and E the set of edges. As every word graph $\mathbf{x}$ also defines the sets N and E, we will use $N(\mathbf{x})$ and $E(\mathbf{x})$ in the remainder to denote the set of nodes and edges of graph $\mathbf{x}$, respectively. Every edge $(x_i, x_j) \in E(\mathbf{x})$ is assigned a positive weight given by a cost function $cost : (x_i, x_j) \mapsto \Re^+$. A path $\mathbf{y}$ in the graph $\mathbf{x}$ is a sequence of connected nodes of $\mathbf{x}$ and the cost of such a path is given by the sum of the edge costs for every edge that is on the path. Given the word graph $\mathbf{x}$, the shortest path problem is finding the path in $\mathbf{x}$ from x_s to x_e with the lowest costs,

$$\operatorname*{argmin}_{\mathbf{y}} \sum_{(x_i,x_y)\in N(\mathbf{x})} y_{ij} cost(x_i, x_{i+1}) \ \text{ s.t. } \ \mathbf{y} \in path(x_s, x_e).$$

There exist many algorithms for computing shortest paths efficiently [Bellman, 1958; Ford, 1956; Dijkstra, 1959]. Usually, these methods are based on relaxation integer programming, where an approximation of the exact quantity is iteratively updated until it converges to the correct solution. Figure 1 shows an example that visualises the shortest path for a compression graph.

3 Learning the Shortest Path

3.1 Representation

To learn the shortest path, we need to draw features from adjacent nodes in the word graph to learn the score of the connecting edge. Let x_i and x_j be connected nodes of the compression graph $\mathbf{x}$, that is $x_i, x_j \in N(\mathbf{x})$ and $(x_i, x_j) \in E(\mathbf{x})$. We represent the edge between x_i and x_j by a feature vector $\phi(x_i, x_j)$. A path in the graph is represented as an $n \times n$ binary matrix $\mathbf{y}$ with $n = |N(\mathbf{x})|$ and elements $\{\mathbf{y}_{ij}\}$ given by $y_{ij} = [[(x_i, x_j) \in path]]$ where $[[z]]$ is the indicator function returning one if z is true and zero otherwise. The cost of using the edge (x_i, x_j) in a path is given by a linear combination of those features which is parameterised by $\mathbf{w}$,

$$cost(x_i, x_j) = \mathbf{w}^\top \phi(x_i, x_j).$$

Replacing the constant costs by the parameterised ones, we arrive at the following objective function (ignoring the constraints for a moment) that can be rewritten as a generalised linear model.

$$\sum_{(x_i,x_j)\in E(\mathbf{x})} y_{ij}\, \mathbf{w}^\top \phi(x_i, x_j) = \mathbf{w}^\top \Phi(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}, \mathbf{y})$$

Given a word graph $\mathbf{x}$, the shortest path $\hat{\mathbf{y}}$ for a fixed parameter vector $\mathbf{w}$ can now be computed by

$$\hat{\mathbf{y}} = \operatorname*{argmin}_{\mathbf{y}} f(\mathbf{x}, \mathbf{y}),$$

where f is exactly the objective of the shortest path algorithm and the argmin consequently computed by an appropriate solver, such as Yen's algorithm [Yen, 1971].

3.2 Learning Shortest Paths with SVMs

In our setting, word graphs $\mathbf{x} \in \mathcal{X}$ and the best summarising sentence $\mathbf{y} \in \mathcal{Y}$ are represented jointly by a feature map $\Phi(\mathbf{x}, \mathbf{y})$ that allows to capture multiple-way dependencies between inputs and outputs. We apply a generalised linear model $f(\mathbf{x}, \mathbf{y}) = \mathbf{w}^\top \Phi(\mathbf{x}, \mathbf{y})$ to decode the shortest path

$$\hat{\mathbf{y}} = \operatorname*{argmin}_{\mathbf{y}} f(\mathbf{x}, \mathbf{y}),$$

where the quality of f is measured by the Hamming loss

$$\Delta_H(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{2} \sum_{(x_i,x_j)\in E(\mathbf{x})} [[y_{ij} \neq \hat{y}_{ij}]]$$

that details the differences between the true $\mathbf{y}$ and the prediction $\hat{\mathbf{y}}$, where $[[\cdot]]$ is again the indicator function from Section 3.1. Using the loss Δ_H, structural support vector machines [Tsochantaridis *et al.*, 2005] minimise the regularised empirical risk

$$\hat{R}[f] = \|f\|^2 + \sum_{i=1}^{m} \Delta_H \left(\mathbf{y}, \operatorname*{argmin}_{\bar{\mathbf{y}}} f(\mathbf{x}, \bar{\mathbf{y}}) \right).$$

It is often beneficial to rescale the induced margin by the loss to implement the intuition that the confidence of rejecting a mistaken output is proportional to its error. Combining everything, we arrive at the following optimisation problem

$$\begin{aligned} \min_{f,\boldsymbol{\xi}} \quad & \|f\|^2 + \sum_{i=1}^{m} \xi_i \\ s.t. \quad & \forall i \forall \bar{\mathbf{y}} \neq \mathbf{y}_i : f(\mathbf{x}_i, \bar{\mathbf{y}}) - f(\mathbf{x}_i, \mathbf{y}_i) \geq \Delta_H(\mathbf{y}_i, \bar{\mathbf{y}}) - \xi_i \\ & \forall i : \xi_i \geq 0 \end{aligned}$$

which can be solved in polynomial time by cutting planes. The idea behind cutting planes is to instantiate only a minimal subset of the exponentially many constrains. This is achieved by decoding for every training sample the shortest path using our current model, if this is not the correct path, then it is added to the constrains and the model is updated. If the decoded path is the correct one, we need to decode the second best path to verify wether the associated margin constraint is fulfilled; if not, the pair is added to the constraints and the model is updated accordingly. Luckily, we do not need to rely on an expensive two-best shortest path algorithm but can compute the most strongly violated constraint directly via the cost function

$$Q(\bar{\mathbf{y}}) = \Delta_H(\mathbf{y}_i, \bar{\mathbf{y}}) - \mathbf{w}^\top \Phi(\mathbf{x}_i, \bar{\mathbf{y}}) + \mathbf{w}^\top \Phi(\mathbf{x}_i, \mathbf{y}_i) \quad (2)$$

that has to be maximised wrt $\mathbf{y}$. The following proposition shows that we can equivalently solve a shortest path problem for finding the maximiser of Q.

Proposition 1 (Loss augmented inference for shortest path problems)**.** *The maximum $\mathbf{y}^*$ of Q in Equation* (2) *can be equivalently computed by minimising a shortest path problem with cost$(x_i, x_j) = y_{ij} + \mathbf{w}^\top \phi(x_i, x_j)$.*

Proof. Omitted for lack of space. □

Given a parameter vector $\mathbf{w}$ and start and end nodes x_s and x_e, respectively, the optimisation of Q can be performed with the following linear program.

$$\begin{aligned}
&\min_{\bar{\mathbf{y}}} \sum_{ij} \left(y_{ij} + \mathbf{w}^\top \phi(x_i, x_j)\right) \bar{y}_{ij} \\
&s.t. \quad \forall k \in N(\mathbf{x})/\{s,t\} : \sum_j \bar{y}_{kj} - \sum_i \bar{y}_{ik} = 0 \\
&\qquad \sum_j \bar{y}_{sj} - \sum_i \bar{y}_{is} = 1 \\
&\qquad \sum_i \bar{y}_{ie} - \sum_j \bar{y}_{ej} = 1 \\
&\qquad \forall (i,j) : \ y_{ij} \le x_{(i,j)} \quad \wedge \quad \forall (i,j) : \mathbf{y}_{ij} \in \{0,1\}
\end{aligned}$$

The first constraint guarantees that every inner node of the path must have as many incoming as outgoing edges, the second line of constraints guarantees the path to start in x_s and, analogously, the third line ensures that it terminates in x_e. The last line of constraints forces the edges of the path $\bar{\mathbf{y}}$ to move along existing paths of $\mathbf{x}$.

4 Empirical Results

In this section, we empirically compare learning shortest paths to traditional unsupervised approaches. To this end, we also deploy a structural perceptron [Collins and Duffy, 2002; Altun *et al.*, 2003] as a special-case of the presented large-margin approach.

4.1 Artificial Data

We generate artificial graphs with $|N| \in \{10, 20, 30, 40\}$ nodes as follows. For every node in a graph, we sample the number of outgoing edges uniformly in the interval $[\frac{|N|}{2}, |N|]$, and for every edge, a receiving node is sampled uniformly from the remaining nodes. To annotate the optimal path we first draw its length uniformly in the interval $[\frac{|N|}{2}, |N|]$ and randomly select the respective number of nodes from N, while enforcing that every edge in the path is included in the graph as well.

To ensure that the optimal path is actually the one with lowest costs, edge features are sampled from a one-dimensional Gaussian mixture distribution, where the generating component is chosen according to whether the respective edge lies on the the shortest path or not. That is, we introduce two Gaussian components $G_{1,2}$, so that costs for edges lying on the shortest path are drawn from

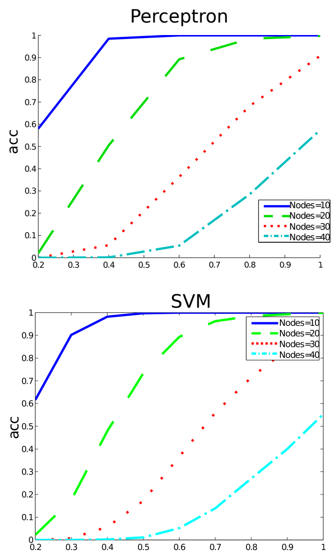


Figure 2: Performance on artificial data for perceptrons (top) and SVMs (bottom).

$G_1(\mu_1, \sigma_1^2)$ while costs for all other edges are sampled from $G_2(\mu_2, \sigma_2^2)$.

The difficulty of the experimental setup is controlled by a parameter α that measures the distance of the two means, i.e., $\alpha = |\mu_1 - \mu_2|$. We sample the means from the following normal distributions $\mu_1 \sim G(-\frac{\alpha}{2}, 0.1)$ and $\mu_2 \sim G(\frac{\alpha}{2}, 0.1)$. We use $\sigma_1 = \sigma_2 = 0.01$ and report on averages over 100 repetitions. The results for perceptrons and SVMs are shown in Figure 2. The distance α is depicted on the x-axis. The y-axis shows the top-one accuracy. The performance of both algorithms highly depends on the distance of the cost-generating components and the size of the graph. Both algorithms perform similarly.

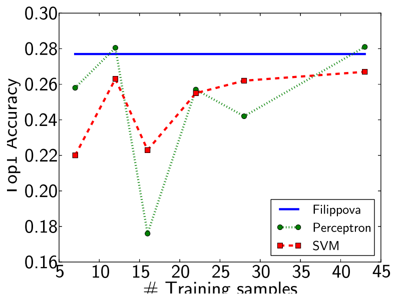
4.2 News Headlines

The real-world data originates from titles of crawled news articles from several web sites on different days. We use categories *Technology, Sports, Business* and *General*. Related sets with more than 4 news headlines are manually identified and grouped together, and word graphs are built according to the procedure described in Section 2.2. Ground truth is annotated manually by selecting the best sentence among the 20 shortest paths computed by Yen's algorithm [Yen, 1971] using frequencies as edge weights. This process leaves us with 87 training examples.

We intend to learn the costs for the edges that give rise to the optimal compression of the training graphs and compare our algorithms to the unsupervised approach presented in [Filippova, 2010] that uses $(\#(x_1) + \#(x_2))/\#(x_1, x_2)$ as edge weights. We devise two different sets of features. The first feature representation consists of only two features that are inspired by the heuristic. That is, for an edge (x_i, x_j), we use

$$\phi_1(x_i, x_j) = \left(\frac{\#(x_1)}{\#(x_1, x_2)}, \frac{\#(x_2)}{\#(x_1, x_2)} \right)^\top,$$

where $\#$ denotes the frequency of nodes and edges, respectively. The second feature representation which is again inspired by [Filippova, 2010] and uses the ingredients of the heuristic instead of precomputing the surrogates to have the



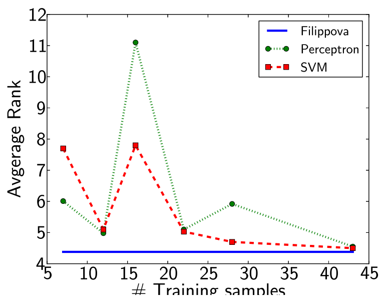


Figure 3: Results on news headlines: Accuracies and average ranks for perceptrons and SVMs.

Table 1: Leave-one-out results for news headlines using feature representation ϕ_2.

	avg. acc.	avg. rank
Filipova	0.277	4.378
Perceptron	0.115	7.58
SVM	0.252	6.942

algorithm pick the best combination,

$$\phi_2(x_i, x_j) = (\#(x_1), \#(x_2), \#(x_1, x_2), \log(\#(x_1)), \log(\#(x_2)), \log(\#(x_1, x_2)))^\top.$$

Figure 3 shows average accuracy (top) and average rank (bottom) for perceptrons and SVMs, respectively, for different training set sizes, depicted on the x-axis. Every curve is the result of a cross-validation that uses all available data. Thus, the rightmost points are generated by a 2-fold cross validation while the leftmost points result from using 11-folds. Due to the small training sets, interpreting the figures is difficult. The unsupervised baseline outperforms the learning methods although there are indications that more training data could lead to better performances of perceptrons and SVMs. The first feature representation shows better performances than the second. However, these conjectures need to be verified by an experiment on a larger scale.

Using only the second feature representation, Table 1 shows average accuracies and average ranks for a leave-one-out setup to increase the sizes of the training sets. The results are promising and not too far from the baseline, however, as before, the evaluation needs to be based on larger sample sizes to allow for interpretations.

5 Conclusion

In this paper, we proposed to learn shortest paths in compression graphs for summarising related sentences. We addressed the previously unsupervised problem in a supervised context and devised structural support vector machines that effectively learn the edge weights of compression graphs, so that a shortest path algorithm decodes the best possible summarisation. We showed that the most strongly violated constrains can be computed directly by loss-augmented inference and rendered the use of two-best algorithms unnecessary. Empirically, we presented preliminary results on artificial and real world data sets. Due to small sample sizes, conclusions cannot be drawn yet, although the results indicate that learning shortest paths could be an alternative to heuristic and unsupervised approaches. Future work will address this question in greater detail.

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